Removal of interference from external coherent signals

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Abstract. We present a technique that we call coherent line removal, for removing external coherent interference from gravitational wave interferometer data. We illustrate the usefulness of this technique applying it to the the data produced by the Glasgow laser interferometer in 1996 and removing all those lines corresponding to the electricity supply frequency and its harmonics. We also find that this method seems to reduce the level of non-Gaussian noise present in the interferometer and therefore, it can raise the sensitivity and duty cycle of the detectors.

INTRODUCTION

In the measured noise spectrum of the different gravitational wave interferometer prototypes [1–3], one observes peaks due external interference, where the amplitudes are not stochastic in contrast to the stochastic noise. The most numerous are powerline frequency harmonics. In this paper we review how to remove these very effectively using a technique we call coherent line removal (CLR) [4,5].

CLR is an algorithm able to remove interference present in the data while preserving the stochastic detector noise. CLR works when the interference is present in many harmonics, as long as they remain coherent with one another. Unlike other existing methods for removing single interference lines [6,7], CLR can remove the external interference without removing any 'single line' signal buried by the harmonics. The algorithm works even when the interference frequency changes. CLR can be used to remove all harmonics of periodic or broad-band signals (e.g., those which change frequency in time), even when there is no external reference source. CLR requires little or no a priori knowledge of the signals we want to remove. This is a characteristic that distinguishes it from other methods such as adaptive noise cancelling [8]. It is 'safe' to apply this technique to gravitational wave data because we expect that coherent gravitational wave signals will appear with at most the fundamental and one harmonic [9]. Lines with multiple harmonics must be of terrestrial origin.

In this paper, we illustrate the usefulness of this new technique by applying it to the data produced by the Glasgow laser interferometer in March 1996 and removing all those lines corresponding to the electricity supply frequency and its harmonics. As a result the interference is attenuated or eliminated by cancellation in the time domain and the power spectrum appears completely clean allowing the detection of signals that were buried in the interference. Therefore, this new method appears to be good news as far as searching for continuous waves (as those ones produced by pulsars [9,10]) is concerned. The removal improves the data in the time-domain as well. Strong interference produces a significant non-Gaussian component to the noise. Removing it therefore improves the sensitivity of the detector to short bursts of gravitational waves [11].

COHERENT LINE REMOVAL

In this section, we summarize the principle of CLR. For further details we refer the reader to [5].

We assume that the interference has the form

$$y(t) = \sum_n a_n m(t)^n + (a_n m(t)^n)^* \ , \tag{1}$$

where a_n are complex amplitudes and $m(t)$ is a nearly monochromatic function near a frequency f_0. The idea is to use the information in the different harmonics of the interference to construct a function $M(t)$ that is

as close a replica as possible of $m(t)$ and then construct a function close to $y(t)$ which is subtracted from the output of the system cancelling the interference. The key is that real gravitational wave signals will not be present with multiple harmonics and that $M(t)$ is constructed from many frequency bands with independent noise. Hence, CLR will little affect the statistics of the noise in any one band and any gravitational wave signal masked by the interference can be recovered without any disturbance.

We assume that the data produced by the system is just the sum of the interference plus noise

$$x(t) = y(t) + n(t) \ , \tag{2}$$

where $y(t)$ is given by Eq. (1) and the noise $n(t)$ in the detector is a zero-mean stationary stochastic process. The procedure consists in defining a set of functions $\tilde{z}_k(\nu)$ in the frequency domain as

$$\tilde{z}_k(\nu) \equiv \begin{cases} \tilde{x}(\nu) & \nu_{ik} < \nu < \nu_{fk} \\ 0 & \text{elsewhere} \ , \end{cases} \tag{3}$$

where (ν_{ik}, ν_{fk}) correspond to the upper and lower frequency limits of the harmonics of the interference and k denotes the harmonic considered. These functions are equivalent to

$$\tilde{z}_k(\nu) = a_k \widetilde{m^k}(\nu) + \tilde{n}_k(\nu) \ , \tag{4}$$

where $\tilde{n}_k(\nu)$ is the noise in the frequency band of the harmonic considered. Their inverse Fourier transforms yield

$$z_k(t) = a_k m(t)^k + n_k(t) \ . \tag{5}$$

Since $m(t)$ is supposed to be a narrow-band function near a frequency f_0, each $z_k(t)$ is a narrow-band function near kf_0. Then, we define

$$B_k(t) \equiv [z_k(t)]^{1/k} \ , \tag{6}$$

that can be rewritten as

$$B_k(t) = (a_k)^{1/k} m(t) \beta_k(t) \ , \qquad \beta_k(t) = \left[1 + \frac{n_k(t)}{a_k m(t)^k} \right]^{1/k} \ . \tag{7}$$

All these functions, $\{B_k(t)\}$, are almost monochromatic around the fundamental frequency, f_0, but they differ basically by a certain complex amplitude. These factors, Γ_k, can easily be calculated, and we can construct a set of functions $\{b_k(t)\}$

$$b_k(t) = \Gamma_k B_k(t) \ , \tag{8}$$

such that, they all have the same mean value. Then, $M(t)$ can be constructed as a function of all $\{b_k(t)\}$ in such a way that it has the same mean and minimum variance. If $M(t)$ is linear with $\{b_k(t)\}$, the statistically the best is

$$M(t) = \left(\sum_k \frac{b_k(t)}{\mathrm{Var}[\beta_k(t)]} \right) \Big/ \left(\sum_k \frac{1}{\mathrm{Var}[\beta_k(t)]} \right) \ , \tag{9}$$

where

$$\mathrm{Var}[\beta_k(t)] = \frac{\langle n_k(t) n_k(t)^* \rangle}{k^2 |a_k m(t)^k|^2} + \text{corrections} \ . \tag{10}$$

In practice, we approximate

$$|a_k m(t)^k|^2 \approx |z_k(t)|^2 \ , \tag{11}$$

and we assume stationary noise. Therefore,

$$\langle n_k(t) n_k(t)^* \rangle = \int_{\nu_{ik}}^{\nu_{fk}} S(\nu) d\nu \ , \tag{12}$$

where $S(\nu)$ is the power spectral density of the noise.

Finally, it only remains to determine the amplitude of the different harmonics, which can be obtained applying a least square method.

REMOVAL OF 50 HZ HARMONICS

In this section, we present experimental results that demonstrate the performance of the CLR algorithm and show its potential value. We apply this method to the data produced by the Glasgow laser interferometer in March 1996 and the electrical interference is successfully removed.

In the study of the Glasgow data, we observe in the power spectrum many lines. Some of them are due to thermal noise (which we will not consider here) and many others at multiples of 50 Hz due external interference, where the amplitudes are not stochastic. In long-term Fourier transforms, the lines at multiples of 50 Hz are broad, and the structure of different lines is similar apart from an overall scaling proportional to the frequency. In smaller length Fourier transforms, the lines are narrow, with central frequencies that change with time, again in proportion to one another. It thus appears that all these lines are harmonics of a single source (e.g., the electricity supply) and that their broad shape is due to the wandering of the incoming electricity frequency.

In the Glasgow data, those lines at 1 kHz have a width of 5 Hz. Therefore, we can ignore these sections of the power spectrum or we can try to remove this interference in order to be able to detect gravitational waves signals masked by them.

In order to remove the electrical interference, we separate the data into groups of 2^{19} points (approximately two minutes) and, for each of them, the coherent line removal algorithm is applied. A detailed description can

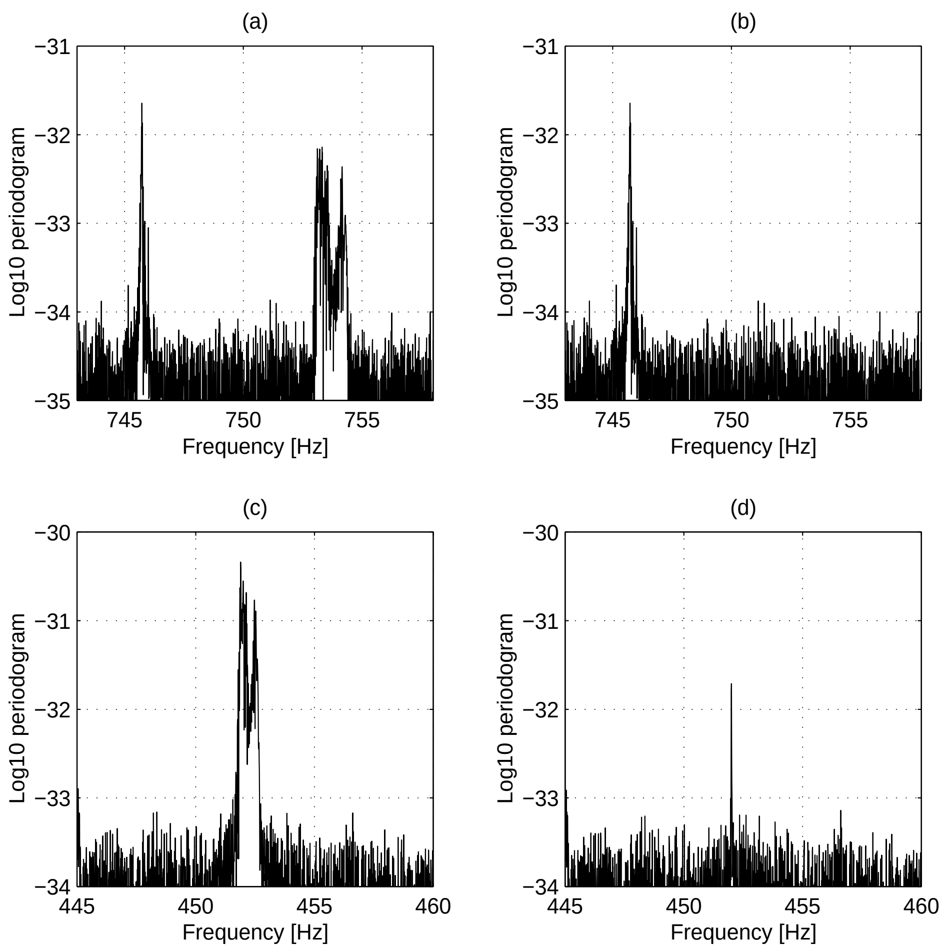

FIGURE 1. Decimal logarithm of the periodogram of 2^{19} points (approximately 2 minutes) of the Glasgow data. (a) One of the harmonics near 754 Hz. (b) The same data after the removal of the interference as described in the text. (c) The same experimental data with an artificial signal added at 452 Hz. (d) The data in (c) after the removal of the interference, revealing that the signal remains detectable. Its amplitude is hardly changed by removing the interference.

be found in [5].

In Fig. 1, we show the performance of CLR on two minutes of data. We can see how CLR leaves the spectrum completely clean of the electrical interference and keeps the intrinsic detector noise. CLR is also applied to the true experimental data with an external simulated signal at 452 Hz, that is initially hidden due to its weakness and we succeed in removing the electrical interference while keeping the signal present in the data, obtaining a clear outstanding peak over the noise level.

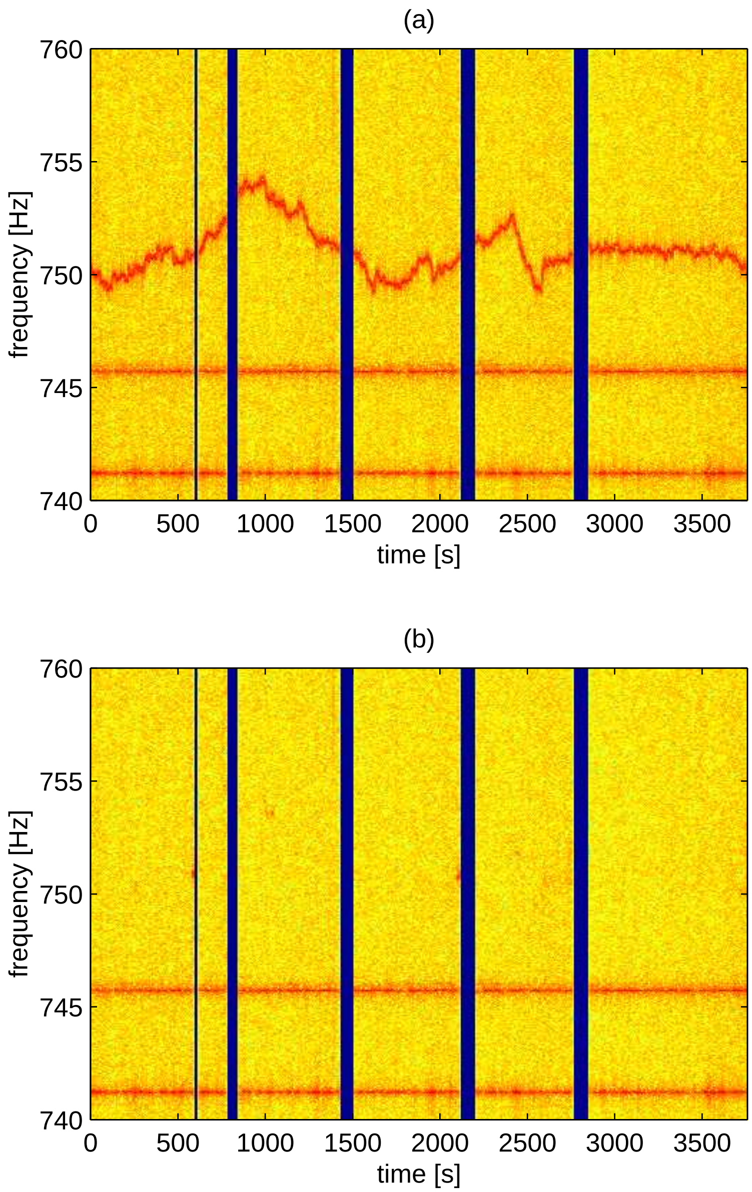

FIGURE 2. Comparison of a zoom of the spectrogram. The dark areas correspond to the periods in which the detector is out of lock. (a) is obtained from the Glasgow data. We can observe the wandering of the incoming electrical signal. The other two remaining lines at constant frequency correspond to violin modes. (b) The same spectrogram as in (a) after applying coherent line removal, showing how the electrical interference is completely removed.

In Fig. 2, we compare a zoom of the spectrogram for the frequency range between 740 and 760 Hz. There we can see the performance of the algorithm on the whole data stream. We show how a line due to an harmonic of the electrical interference in the initial data is removed.

We are interested in studying possible side effects of the line removal on the statistics of the noise in the time domain. We observe that the mean value is hardly changed. By contrast, a big difference is obtained for the standard deviation. For the Glasgow data, its value is around 1.50 Volts. After the line removal, the standard deviation is reduced, obtaining a value around 1.05 Volts. This indicates that a huge amount of power has been removed.

Further analysis reveals that values of skewness and kurtosis are getting closer to zero after the line removal. Values of skewness and kurtosis near zero suggest a Gaussian nature. Therefore, we are interested in studying the possible reduction of the level of non-Gaussian noise. To this end, we take a piece of data and we study their histogram, calculating the number of events that lie between different equal intervals. If we plot the logarithm of the number of events versus $(x-\mu)^2$, where x is the central position of the interval and μ is the mean, in case of a Gaussian distribution, all points should fit on a straight line of slope $-1/2\sigma^2$, where σ is the standard deviation. We observe that this is not the case (see figure 3). Although, both distributions seem to have a linear regime, they present a break and then a very heavy tail. The two distributions are very different. This is mainly due to the change of the standard deviation.

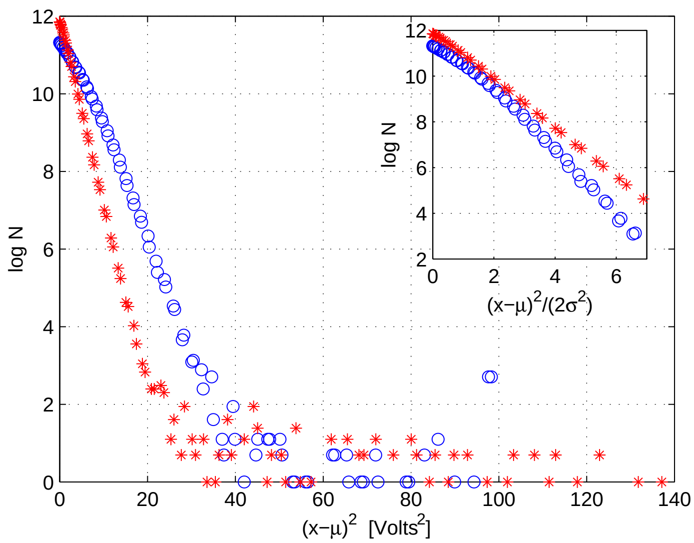

FIGURE 3. Comparison of the logarithm plot of the histogram for 3.2×2^{19} points as a function of $(x-\mu)^2$. The circles correspond to the Glasgow data and the stars to the same data after removing the electrical interference. The Glasgow data is characterized by $\mu = -0.0182$ Volts and $\sigma = 1.5151$ Volts. After the line removal, we obtain the values of $\mu = -0.0182$ Volts and $\sigma = 1.0449$ Volts. In the right-hand corner, there is zoom of the original figure, but rescaled so that the abscissa corresponds to $(x-\mu)^2/2\sigma^2$. If the data resembles a Gaussian distribution, we will expect a single straight line of slope -1. This is not the case for the Glasgow data, but it seems to be satisfied for the clean data up to 4σ. The large number of points in the highest bin of the Glasgow data is an effect of saturating the ADC. These points are spread to higher and lower voltages by line removal.

We can zoom the 'linear' regime and change the scale in the abscissa to $(x-\mu)^2/(2\sigma^2)$. Then, any Gaussian distribution should fit into a straight line of slope -1. We observe that after removing the interference, it follows a Gaussian distribution quite well up to 4σ. The original Glasgow data does not fit a straight line anywhere.

In order study the Gaussian character, we have also applied two statistical tests to the data: the chi-square test that measures the discrepancies between binned distributions, and the one-dimensional Kolmogorov-Smirnov test that measures the differences between cumulative distributions of a continuous data.

We computed the significance probability for every 2^{12} points of the data using both tests and we checked whether the distribution are Gaussian or not. The two tests are not equivalent but in any case, the values of the significance probability would be close to unity for distributions resembling a Gaussian distribution. In both tests, the significance probability increased after removing the electrical interference, showing that this procedure suppresses some non-Gaussian noise, although, generally speaking, the distribution was still

non-Gaussian in character, presumably because of the heavy tails, which are not affected by line removal. See [5] for details.

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